Tentamen Metrische Ruimten, 28/06/04

1. Let B be a totally bounded subset of a metric space M. Show that B is bounded. Give an example of a bounded metric space which is not totally bounded.

2. Let A be an infinite set with some element $p \in A$. Let $\mathfrak{T}$ consist of all subsets U of A such that $p \in U$, together with the empty set $\emptyset$.

   (a) Show that $T = (A, \mathfrak{T})$ is a topological space.

   (b) Is T compact?

   (c) Is T connected?

   (d) Is T Hausdorff?

   (e) Can $\mathfrak{T}$ be generated by a metric defined on A?

   Justify the answers!

3. Let the topological space T be Hausdorff. Show that the finite subsets of T are closed.

4. Determine the closure and the boundary of each of the following subsets of $\mathbb{R}$ with the usual Euclidean metric. Which of these sets are dense or nowhere dense in $\mathbb{R}$? ($\mathbb{Q}$ is the set of rational numbers, $\mathbb{Z}$ the set of integers, $\mathbb{N}$ the set of positive integers.)

   (a) $\mathbb{R}$;

   (b) $\mathbb{Q} \cap (-\infty, 0)$;

   (c) $\{\frac{7}{3n} : n \in \mathbb{N}\}$;

   (d) $\mathbb{R} \setminus \mathbb{Q}$;

   (e) $\mathbb{R} \setminus \mathbb{Z}$;

   (f) $\mathbb{N}$.